SPATIAL DISTRIBUTION OF DIELECTRIC RADIATION

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Various factors contributing to the nonisotropic character of surface dielectric radiation are considered. The error arising from the assumption of isotropic radiation is estimated in a number of cases.

In engineering calculations, one assumes the radiation from the surfaces of bodies to be isotropic. This assumption has been put to test on a number of occasions. Calculations of the angular coefficients for the surfaces of metals have been made [1]. It was found that the assumption of isotropic radiation causes an error of as much as 24%. Hering [2] calculates the heat exchange between infinite gray strips forming a dihedral angle. The reflection of rays agrees with electromagnetic theory. It has been established that the assumption of constant optical properties in the directions of specular reflection is acceptable for engineering calculations. Radiation from dielectrics is more isotropic than that from metals. However, even in the case of dielectrics one must estimate the error due to the anisotropy of the optical constants. We obtain this estimate for equilibrium radiation by solving the simplest classical problem. We then consider the case of nonequilibrium dielectric radiation.

The theoretical expression for the coefficient of absorption of a smooth dielectric is

$$a(\mu,\ n) = 1 - \frac{1}{2}\left[\left(\frac{\mu - \sqrt{n^2 - 1 + \mu^2}}{\mu + \sqrt{n^2 - 1 + \mu^2}}\right)^2 + \right.$$
$$\left. + \left(\frac{n^2\mu - \sqrt{n^2 - 1 + \mu^2}}{n^2\mu + \sqrt{n^2 - 1 + \mu^2}}\right)^2\right],$$

which is in good agreement with experiment [3, 4]. For an isotropic incident flux, the coefficient of absorption (averaged over polarizations and angles of incidence) depends only on the index of refraction:

$$A(n) = 2\int_0^1 a(\mu,\ n)\,\mu\,d\mu.$$

Using this formula in our calculations, we verified the curve of the coefficient of absorption given in Figs. 2–6 of [4]. It should be noted that this integral can be expressed in terms of elementary functions [5].

In the current theory of radiant energy transfer, a plane layer of gas contitutes the classical model. We therefore consider various transmittivities of a plane-parallel layer for isotropic and real equilibrium radiation of the dielectric which forms the boundary of the medium. The transmittivity

$$D(\tau,\ n) = \frac{2}{A}\int_0^1 \exp\left(-\frac{\tau}{\mu}\right) a(n,\ \mu)\,\mu\,d\mu.$$

If the radiation of the surface is isotropic, $A = a$, and

$$D_0(\tau) = 2\int_0^1 \exp\left(-\frac{\tau}{\mu}\right)\mu\,d\mu = 2E_3(\tau).$$

As a measure of the isotropy of the surface radiation, we take the ratio

$$\delta = D(\tau,\ n)/D_0(\tau).$$

Table 1 shows that for equilibrium radiation of dielectrics there is an enhanced transmittivity (up to 5%) as a result of a somewhat elongated indicatrix with maximum error at n = 1.7. A series was used to calculate accurately the function $E_3(\tau)$, making the computational error negligible. The integral was replaced by a Gauss quadrature with seven nodes. In defining the angular coefficients for two arbitrary surfaces, the analogous error for radiation into a solid angle smaller than, say, 2π, can be larger. On the whole, however, the assumption of isotropic radiation is applicable in engineering calculations for states of bodies near equilibrium. In the more common case, the surface forming the boundary of the layer is translucent. Table 2 gives the coefficients of absorption for several non-turbid materials. Thin plates made of these materials have optical thicknesses close to unity. In calculations of their radiation or transmission, one must take multiple reflections inside the plate into account. The indicatrices of the

Table 1
Ratio δ of Transmittivities of a Layer of Gas with Optical Thickness τ, for Real and Isotropic Dielectric Radiation. n = Index of Refraction

n	τ = 0.1	0.5	1.0	2.0
1.1	1.0085	1.0171	1.0203	1.0221
1.2	1.0113	1.0249	1.0306	1.0343
1.3	1.0128	1.0293	1.0367	1.0419
1.5	1.0141	1.0335	1.0426	1.0494
1.7	1.0143	1.0346	1.0443	1.0517
2.0	1.0139	1.0334	1.0429	1.0502
3.0	0.1102	1.0224	1.0279	1.0318

characteristic and transmitted radiation of an isothermal dielectric of finite optical thickness was studied [6] for layer thicknesses larger than the wavelength. The curves obtained show that the spatial distribution of the radiation depends very little on the optical thickness. For n = 1 (in the case of gases) the calculation is fairly simple; it was done by Gershun [7], and a contrary conclusion may be drawn. We therefore assume the effect of the optical thickness to be weak only for condensed dielectrics for which the reflection is effectively internal.

We know of many attempts to calculate the scattering inside a dielectric (for example, in radiation transfer in milky glasses and similar materials [7]), but these attempts were rather crude and were done for bodies without characteristic radiation. An accurate calculation (for n = 1) is fairly complicated [8].

A real plate is rough. A detailed calculation [5] showed that the effect of the roughness of the dielectric on the coefficient of absorption is negligible. This theoretical conclusion has been experimentally [9] verified.

Finally, we must take into account the anisotropy of the dielectric radiation in the presence of varying temperature field. Usually, the resultant energy flux passes through the surface and then the temperature gradient appears. The intensity of the characteristic surface radiation is approximately described by the formula

$$I(\mu) = \frac{a(\mu)}{n^2}\left[\int_0^{\tau_0} B(\tau)\exp\left(-\frac{\tau}{\mu_1}\right) d\left(\frac{\tau}{\mu_1}\right) + B_e \exp\left(-\frac{\tau_0}{\mu_1}\right)\right],$$

$$\mu_1 = \sqrt{n^2 - 1 + \mu^2}/n. \quad (1)$$

The function $a(\mu)$ is assumed to be independent of the optical thickness and the roughness in accordance with the work described above. It depends on the field temperature via the function n(T). However, we also neglect this dependence and calculate $a(\mu)$ for a certain average temperature. This function may, in fact, be replaced by an average coefficient of absorption A, because the effect of the argument μ (in view of the non-uniformity of the temperature) may be enormously larger; our task is to estimate this effect. To determine the indicatrix of the characteristic surface radiation, we use the formula

$$\Phi(\mu) = I(\mu)/I(1) = \left[\int_0^{\tau_0} B(\tau)\exp\left(-\frac{\tau}{\mu_1}\right) d\left(\frac{\tau}{\mu_1}\right) + B_e \exp\left(-\frac{\tau_0}{\mu_1}\right)\right] \Big/ \left[\int_0^{\tau_0} B(\tau)\exp(-\tau)\,d\tau + B_e\exp(-\tau_0)\right].$$

It is simple to include in the analysis the case of negligible scattering in the body, assuming gray radiation and local thermodynamic equilibrium. We then obtain

$$B(\tau) = n^2 \sigma T^4/\pi.$$

We can represent the temperature field by the power series

$$T = a_* + b_* \tau + \ldots.$$

In this connection we express $I(\mu)$ and $\Phi(\mu)$ as a series of cosine terms, since these are the most suitable for calculating angular coefficients and other characteristics of the radiative heat transfer which are external for the given body [10]. For very large absorption coefficients, such as those of metals, the numbers b_*, c_*, ... are nearly zero, and the temperature field of the region from which the surface radiation enters is practically homogeneous under all conditions. The surface radiation will be isotropic. Most dielectrics will, however, transmit to a certain extent for thermal conductivities much smaller than in the case for metals. According to Table 2, silicon and artificial sapphire—materials used in deflectors of infrared radiation collectors—have absorption coefficients on the order of 20 m^{-1} and 200 m^{-1}. They transmit ≈99% of the characteristic surface radiation from an adjacent layer as much as 0.2 and 0.02 m thick, respectively. A large temperature gradient appears when there is intense heat transfer in this layer. As an example, let us determine the indicatrix of the characteristic radiation of a plate of optical thickness $\tau_0 \simeq 4.5$. We may take $\tau_0 = \infty$ with an error of less than 1%. For silicon and sapphire, the thickness will be less than 0.2 and 0.02 m, respectively. For a linear temperature field, we have

$$B(\tau) = n^2\sigma(a_* + b_*\tau)^4/\pi.$$

According to formula (1),

$$\Phi(\mu) = \frac{1 + 4b\mu_1 + 12b^2\mu_1^2 + 24b^3\mu_1^3 + 24b^4\mu_1^4}{1 + 4b + 12b^2 + 24b^3 + 24b^4};$$

$$b = \frac{b_*}{a_*}.$$

For $b \gg 1$, $\Phi(\mu) \simeq \mu_1^4 = [(n^2 - 1 + \mu^2)/n^2]^2$, whereas $\Phi(\mu) = 1$ for isotropic radiation.

For the specific radiation intensity used in [10], the indicatrix equals

$$f(\mu) = \Phi(\mu)\mu.$$

Table 2
Coefficients of Absorption of Several Materials. Temperatures Indicated Only for Silicon

Material	k[m^{-1}] (λ, μ; t, °C)
Silicon	32 (3; 25)
	70 (3; 350)
	17 (6; 25)
	21 (6; 350)
Artificial sapphire	190 (5.35)
	760 (6.3)
MgO (fused)	41 (0.5)
	38 (5)
MgF_2 (crystalline)	20 (5.3)

The equivalent solid angle [10] is

$$\Omega = 2\pi \int_0^1 f(\mu)\, d\mu.$$

Using the general formulas [10, 11], we can now find the angular coefficients and other characteristics of the heat transfer.

The form of Eq. (1) most suitable for a gas ($n = 1$, $a(\mu) = A = 1$) is used. In this case the thermal conductivity is negligible. Because of the absence of convection, the radiation mechanism defines the temperature field, and $B(\tau)$ is nearly linear. If the radiation is in vacuum, the intensity of the surface radiation of a body equals [12]

$$I(\mu) = \frac{3q}{4\pi}\left(C_1 + \mu - \frac{C_2}{3.7\mu + 1}\right),$$

which is a good description of the best results of [13]. The deviation from the data points of [14] is at most 1.2%. According to (1),

$$\Phi(\mu) = \left(C_1 + \mu - \frac{C_2}{3.7\mu + 1}\right)\left(1 + C_1 - \frac{C_2}{4.7}\right)^{-1},$$

where $C_1 = 0.7104$ and $C_2 = 0.1331$.

For isotropic radiation, $\Phi(0) = 0.343$ replaces $\Phi(\mu) = 1$, since the intensity along the surface is almost 1/3 that along the perpendicular. Equation (2) loses its meaning for the case of thermodynamic equilibrium ($q = 0$). This would imply zero temperature. For real boundary conditions on the surface the radiation flux comes in from the outside, which is equivalent to a medium temperature T outside the surface given by $\sigma T^4/\pi = I_0$. In such a case the indicatrix equals

$$\Phi(\mu) = \left[I_0 + \frac{3q}{4\pi}\left(C_1 + \mu - \frac{C_2}{3.7\mu + 1}\right)\right] + \left[I_0 + \frac{3q}{4\pi}\left(C_1 + 1 - \frac{C_2}{4.7}\right)\right]^{-1}.$$

It is clear that this radiation is isotropic only for $I_0 \gg q/\pi$, i.e., for quasi-equilibrium conditions.

To summarize: our analysis has shown that the moderate forward peaking of the indicatrix of the characteristic radiation from dielectrics may be strongly enhanced if there is heat transfer from the body but will be flattened out if there is substantial external heat input. The error we get from assuming isotropic radiation may appear to exceed tolerable limits. If we know the optical properties of the body and the temperature field of the body near the surface, we can estimate the error by using the formulas given previously. These results must be used, in particular, in making experimental determinations of the degree of blackness for materials having large amounts of energy takeoff from their surfaces.

NOTATION

$\mu \equiv \cos\theta$; θ is the angle between normal to surface and ray; μ_1 is the same for ray inside dielectric; n is the refraction index; $a(\mu, n)$ is the absorption coefficient for direction defined by μ; $A(n)$ is the mean hemispherical absorption coefficient; $D(\tau, n)$ is the transmittivity of gas layer for real dielectric radiation (probability that a quantum of incident flux energy will penetrate layer without interacting with medium); $D_0(\tau)$ is the same for isotropic radiation incident on layer; $\tau = \int_l kdl$ is the optical depth of layer; k is the attenuation coefficient, m^{-1}; l is the distance along normal from surface, m; τ_0 is the optical thickness; $\delta = D/D_0$; q is the resultant radiation flux in dielectric, W/m^2; $I(\mu)$ is the intensity of characteristic surface radiation, W/m^2-sr; B is the "yield" of volume element (for gray radiation at local thermodynamic equilibrium and infinitesimal scattering, $B = n^2\sigma T^4/\pi$); σT^4 is the density of hemispherical black radiation; B_e is the effective "yield" of an element of surface bounding plate from the other side (intensity of flux reflected from it); $\Phi(\mu)$ is the indicatrix for the intensity of external radiation of plate; I_0 is the intensity of flux incident on surface from outside; a_* and b_* are the coefficients of series; Ω is the equivalent solid angle [10], sr; λ is the wavelength; t is the temperature, °C.

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